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Abstract

Visco-resistive MHD computations with the NIMROD code [J. Comput. Phys. 195 355 (2004)] are applied to a model tokamak configuration that is subjected to induced vertical displacement. The modeling includes anisotropic thermal conduction within an evolving magnetic topology, and parameters separate the Alfvénic, resistive-wall, and plasma-resistive timescales. Contact with the wall leads to increasingly pervasive kink and tearing dynamics. The computed 3D evolution reproduces distinct thermal-quench and current-quench timescales, a positive bump in plasma current, and net horizontal forcing on the resistive wall. The MHD dynamo effect electric field, \( E_f = -\langle \vec{V} \times \vec{B} \rangle \), is analyzed for understanding the nonlinear effects of the fluctuations on the spreading of parallel current density and the resulting bump in plasma current. Forces on the resistive wall are consistent with Pustovitov's analysis [Nucl. Fusion 55 113032 (2015)]; the plasma remains in approximate force-balance with the wall, so net force is accurately computed from integrating stress over the wall's outer surface. Improvements to the modeling that are needed for predictive simulation of asymmetric vertical displacement events are discussed.

1. Introduction

Discharge-terminating disruptive events in tokamaks take many forms and involve a variety of plasma dynamics [1, 2]. Whether it is a root cause or a consequence of other disruptive activity, vertical displacement poses a severe risk in large experiments, because it brings hot plasma in contact with surfaces that are not designed for extreme thermal loading. In addition, asymmetries that develop during vertical displacement events (VDEs) lead to horizontal forces on electrically conducting structures [3-6], and the forcing may rotate at rates that are comparable to mechanical harmonics [7]. The risks for ITER have motivated numerous experimental, analytical, and computational studies of disruptive dynamics and of means to mitigate their consequences. Here, we report on nonlinear visco-resistive magnetohydrodynamic (MHD) computations of vertical displacement in a model configuration and on the consequences of significant asymmetry that develops through contact with the wall. In these computations, vertical displacement occurs over the timescale of the resistive wall and is forced by externally imposed conditions on the magnetic field. Our analysis examines the force on the resistive wall during the current quench (CQ) and the spreading of parallel current density, which transiently raises the plasma current starting at the thermal quench (TQ).

To date, simulations of disruptions have been based on MHD modeling or on forms of reduced MHD [8-17]. The reasoning behind this choice is that macroscopic dynamics always arise during disruptions, and MHD provides a practical model for dynamics involving gross motions and changes in magnetic topology. Because disruptions in tokamak experiments involve electron and ion kinetics, radiation, neutral dynamics, and plasma-surface interaction, in addition to macroscopic dynamics, predictive simulation of disruptions in tokamaks will require far more comprehensive modeling [18] that is beyond present-day capabilities. Nonetheless, MHD
modeling provides a basis for understanding disruptive dynamics and a conceptual framework from which to build comprehensive modeling. Previous axisymmetric studies evolve equilibrium relations quasi-statically to examine the generation of symmetric, open-field halo currents [11] and the influence of geometric details of surrounding structures [19]. Previous simulations of asymmetric VDEs by Strauss and coauthors examined the peaking of plasma current over the toroidal angle [15], the magnitude of horizontal forces as the ratio of external-kink time and resistive-wall time is varied [6], and the influence of changing boundary conditions on flow [20]. Recent simulations of a VDE in the NSTX experiment describe the destabilization of edge kink modes resulting from contact with solid surfaces [21].

Experimental studies of VDEs trigger them by turning parts of the control system off [22] or by programming the system to induce displacement [23]. The computations discussed here are similar to the latter approach. They start from a nominally up-down symmetric, double-null equilibrium, and one of the two divertor coils is effectively turned off at the start of the computation. Vertical displacement then proceeds on the resistive wall time $\tau_w$, as eddy currents decay, which is the case whenever the wall slows the displacement. Running both axisymmetric and 3D nonlinear computations in this controlled scenario allows direct comparison, which helps us identify effects that result from asymmetry.

The remaining sections of this paper are organized as follows. Section 2 briefly describes the model and computational methods, including parameters and initial conditions. Section 3 discusses the computed results on displacement, TQ, CQ, and asymmetries. It also describes analysis of the spreading of parallel current density and forces on the wall. Section 4 provides a discussion of our findings and future efforts, and our conclusions are given in Section 5.

2. Visco-resistive MHD modeling

The central objective of this study is to reproduce the major aspects of asymmetric disruption from vertical displacement. Modeling a TQ resulting self-consistently from asymmetric macroscopic instability requires thermal transport that is sensitive to changes in magnetic topology. The CQ occurs over a longer time-scale in experiments [1, 24-26], so numerical computations must also have multi-scale capabilities. In addition, net forcing on the wall only results when the wall is not an ideal conductor [27]. All of these properties are within the scope of time-dependent nonlinear non-ideal MHD models if the system of equations is solved with implicit numerical methods and is augmented with a resistive-wall and external magnetic response.

We separate the problem domain, which is axisymmetric, into the two subdomains shown in Fig. 1, where the resistive wall lies along their intersection. The visco-resistive MHD equations of the inner subdomain describe the evolution of particle density $(n = n_e = n_i)$, plasma flow velocity $(\mathbf{V})$, a single plasma temperature $(T = T_e = T_i)$, and magnetic field $(\mathbf{B})$:

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot \left( D_n \nabla n - D_h \nabla^2 n \right),
\]

(1)
\[ mn \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V} = \mathbf{J} \times \mathbf{B} - \nabla P - \nabla \cdot \mathbf{\Pi}, \quad (2) \]

\[ \frac{1}{\Gamma - 1} n \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) T = -n T \mathbf{V} \cdot \nabla - \mathbf{q}, \quad (3) \]

\[ \frac{\partial}{\partial t} \mathbf{B} = -\nabla \times (\eta \mathbf{J} - \mathbf{V} \times \mathbf{B}) + \kappa_b \nabla \nabla \cdot \mathbf{B}, \quad (4) \]

where total (electron plus ion) pressure is \( P = 2nT \), \( \mu_0 \mathbf{J} = \nabla \times \mathbf{B} \), and \( \Gamma = 5/3 \) is the adiabatic index. Relative to SI units, the Boltzmann constant is absorbed in \( T \). The computations normalize the fields by using dimensions, ion mass \( m \), maximum-\( n \), and \( |\mathbf{B}| \) of order unity and by setting \( \mu_0 = 1 \).

Figure 1. Poloidal cross-section of the domain for the forced VDE computations. The red line indicates the location of the resistive wall, which surrounds the inner subdomain that contains plasma. The black line indicates the location of a conducting shell that surrounds the outer subdomain. The small rectangles indicate the locations of the 15 external coils.

The first term on the right side of Eq. (4) represents induction from the resistive-MHD electric field. We use the Spitzer dependence of \( \eta = \eta_0 \left( T_0 / T \right)^{3/2} \) with \( \eta_0 = 1 \times 10^{-6} \) in normalized units where \( \tau_A = R_0 \sqrt{\mu_0 mm_0 / B_0} = 1 \). The initial state, described below, has \( T \) varying by four orders of magnitude from the open-field region to the central value \( T_0 \) of the initial equilibrium. Large resistivity at low temperature suppresses current density outside regions representing hot plasma during the course of 3D dynamics. The second term on the right side of Eq. (4) is a numerical

\(^1\text{More precisely, conditions at the magnetic axis of the equilibrium discussed below have } R_0=1.65 \text{ and } B_0=1.28, \text{ so the normalized } \tau_A \text{ is 1.29. For convenience, discussion in the text considers } \tau_A \text{ to be 1, however.}\)
term that, together with a high-order representation, is used to keep magnetic divergence error small in the computations [28]. The terms on the right side of Eq. (1) provide smoothing of the particle density field with the relatively small coefficients of \( D_n = 5 \times 10^{-6} \) and \( D_n = 1 \times 10^{-10} \). The closure relations represent anisotropic thermal conduction

\[
q = -\eta \left( (\chi_\parallel - \chi_\perp) \hat{b} \cdot \nabla T + \chi_\perp \nabla T \right) 
\]

and anisotropic viscous stress

\[
\Pi = -\eta \left[ (\nu_\parallel - \nu_\perp) \left( 3 \hat{b} \hat{b} - 1 \right) \hat{b} \cdot \nabla \hat{b} + \nu_\perp \hat{W} \right],
\]

where \( I \) is the identity tensor, and \( \hat{W} = \nabla \hat{V} + \nabla \hat{V}^T - (2/3) I \hat{V} \cdot \hat{V} \) is the traceless rate of strain tensor. While the form of these relations is, technically, only suitable for collisional plasma with vanishingly small ion gyro-radii [29], it facilitates our present computations, together with the relatively modest anisotropy from the coefficients \( \chi_\perp = 5 \times 10^{-6} \), \( \chi_\parallel = 5 \times 10^{-2} \), \( \nu_\parallel = 5 \times 10^{-5} \), and \( \nu_\perp = 5 \times 10^{-2} \). The unit vector \( \hat{b} \) is along the 3D magnetic field, which makes transport sensitive to the evolving magnetic topology, even with the modest anisotropy.

The outer subdomain is modeled by a simplified version of Eq. (4), where the electric field is \( \mathbf{E} = \eta \mathbf{J} \) with \( \eta \) having the fixed value of 100 so that resistive diffusion there is faster than other processes in the problem. This annular subdomain lies between the resistive wall and an outer conducting shell. The two subdomains are coupled by the thin-wall approximation of

\[
\frac{\partial (\mathbf{B} \cdot \hat{n})}{\partial t} = -\hat{n} \cdot \nabla \times \left[ \nu_w \hat{n} \times \Delta \mathbf{B} \right],
\]

where \( \hat{n} \) is the unit normal along the surface of the wall, the parameter \( \nu_w = \eta_w / \mu_0 \Delta_w \) is the ratio of the wall's magnetic diffusivity and thickness, and \( \Delta \mathbf{B} \) is the jump in magnetic field across the wall. For spatial scales of order unity, our \( \nu_w \)-value of \( 10^{-3} \) implies that the time-scale for diffusion through the wall is \( \tau_w \sim 10^3 \tau_A \). The initial central-\( \eta_0 \) value implies \( \tau_{\eta} \sim 10^3 \tau_w \) over the same spatial scale, hence \( \tau_{\eta} >> \tau_w >> \tau_A \).

The conducting shell that surrounds the outer subdomain holds the normal component of \( \mathbf{B} \) constant. The axisymmetric part of \( \mathbf{B} \cdot \hat{n} \) along that surface has contributions from external coils and from the initial plasma current. In addition, small error fields of order \( 10^{-7} \) are imposed on toroidal harmonics \( n = 1 \) and \( n = 2 \) at this surface. Equations (1-3) are only solved in the inner region, so their boundary conditions are imposed along the resistive wall. Inhomogeneous Dirichlet conditions for \( n \) and \( T \) maintain the initial edge values of \( n = 0.1 n_0 \) and \( T = 10^{-4} T_0 \), respectively, and homogeneous Dirichlet conditions on \( \mathbf{V} \) set all components of flow velocity to 0. Mass and thermal energy escape the inner subdomain via diffusion and thermal conduction.
Consistent with findings reported in Ref. [20], the concerns regarding the impenetrable flow condition raised in Ref. [30] are not realized in these non-ideal computations, where magnetic flux is not frozen to cooled regions of plasma. However, we have found that results are sensitive to boundary conditions on $T$, due to its impact on resistivity in open-field regions [31]. More realistic modeling with boundary conditions inferred from sheath effects is under development, but it is beyond the scope of the present work and will be presented in a future publication.

The nonlinear computations start from the model equilibrium whose pressure and poloidal flux distribution are shown in Fig. 2a. The equilibrium is the result of solving the Grad-Shafranov equation with the pressure and poloidal current $I = RB\phi$ profiles within the closed-flux region being

$$\mu_0 P(\bar{\psi}) = P_e + (P_1 - 4P_2\bar{\psi})(1 - \bar{\psi}) \quad \text{and} \quad I = I_e + (I_1 - 4I_2\bar{\psi})(1 - \bar{\psi}) \quad \text{(8a)}$$

where $\bar{\psi}$ is the normalized poloidal flux function that increases from 0 at the magnetic axis to 1 at the separatrix. The parameters $P_1 = 1 \times 10^{-2}$, $P_2 = 2 \times 10^{-3}$, $P_e = 1 \times 10^{-7}$, $I_1 = 0.122$, $I_2 = 1 \times 10^{-2}$, and $I_e = 2$ produce the pressure and safety-factor profiles that are shown in Fig. 2b. The equilibrium is determined numerically with the NIMEQ code [32], which has been modified to solve free-boundary computations. This up-down symmetric configuration has 15 axisymmetric coils outside the resistive wall at the positions shown in Fig. 1, and the two divertor coils are located at $R = 1.25$ and $Z = \pm 1.45$. The pressure distribution has a central $\beta$-value of 1.2%, and the particle density profile satisfies $n = n_0 (P/P_e)^{1/5} \quad \text{so that edge-} n$ is 1/10th of its central value.

The results section describes three types of initial-value computations for this study. All are solved with the NIMROD code [28] using the implicit leapfrog time-advance that is described in Ref. [33] with the stabilization scheme from Ref. [34]. They use the same mesh over the $R-Z$ plane, which has approximately 40,000 biquadratic elements in the inner subdomain and approximately 20,000 in the outer subdomain with some degree of concentration near the resistive wall in both. Results computed with the same mesh having bicubic basis functions are similar, providing evidence of spatial convergence over the $R-Z$ plane, but running these computations through the entire current quench is computationally prohibitive at present.

Toroidal variations in 3D NIMROD computations are represented by finite Fourier series. The resolution of toroidal wavenumbers $0 \leq n \leq 21$ in the 3D computation described here is based on experience with scanning resolution over a series of computations. Numerical quadratures over the toroidal angle use 64 evenly spaced points to preclude aliasing of quadratic nonlinearities. This computation includes additional numerical damping of the $n = 20$ and $n = 21$ components at rates of $5 \times 10^{-3}$ and $1 \times 10^{-2}$, respectively, to help avoid aliasing from high-order nonlinearities.
Figure 2. Equilibrium distributions of a) pressure (color) and poloidal flux (lines) and b) profiles of pressure and safety factor ($q$) within the closed-flux region. The $q$-plot trace is limited to normalized flux values of less than unity.

Growth-rates for the initial configuration are computed from the linearized versions of Eqs. (1-4), where the equilibrium is a fixed distribution. In the nonlinear computations, the equilibrium density and temperature distributions become the initial conditions for $n$ and $T$ for the axisymmetric part of the solution. The axisymmetric part of the magnetic field is decomposed; field from the plasma current and from the upper divertor coil, the unfilled rectangle in Fig. 1, become part of the initial conditions, and field from the other 14 coils is held constant. This effectively turns the upper divertor coil off, leaving eddy current in the resistive wall in place of the upper divertor coil current. Also, the nonlinear computations are not driven from loop voltage or other external sources, and even without vertical displacement, this would lead to a gradual decay of plasma current and thermal energy from the start of the computations.

3. Results

For completeness, we first discuss the linear stability properties of the initial state without the forced VDE. We then describe general properties of the nonlinear evolution of axisymmetric and 3D computations with the forced displacement. Analysis of the current density and forces on the resistive wall follow.

3.1. Linear stability of the initial state

The equilibrium profile shown in Fig. 2 is linearly unstable to an external $m=4$, $n=1$ mode that has a growth rate of $4 \times 10^{-2} \tau_A^{-1}$. The eigenfunction plot, Fig. 3, shows that the mode perturbs the edge of the closed-flux region. This is consistent with the fact that the $q$-profile (Fig. 1b) only approaches the value of 4 very close to the separatrix. The equilibrium profiles given by Eqs. (8a-b) produce nonzero parallel current density, meaning $\lambda = \mu_0 J_\parallel / B$, just inside the separatrix, which can be inferred by noting that
\[ \lambda = -I' - \frac{\mu_0 I R^2 \rho'}{|\nabla \psi|^2 + I^2} \]

is nontrivial for \( \bar{\psi} \rightarrow 1 \) in the equilibrium. Near the separatrix of this equilibrium, the above relation is dominated by the first term on the right. The fact that the mode is distributed over the poloidal angle indicates external-kink behavior, likely associated with this edge current density and its discontinuity across the separatrix. In our nonlinear computations, the relatively cool edge region just inside the separatrix is subject to resistive diffusion. However, as discussed in the next section, contact with the wall that results from vertical displacement tends to re-sharpen the edge.

![Figure 3. Color contours of constant pressure for the linearly unstable \( n = 1 \) eigenmode of the initial equilibrium. Poloidal phasing is arbitrary, and the real part of the eigenfunction is plotted.](image)

### 3.2. Nonlinear evolution

The vertical displacement imposed by the decay of eddy current transitions the configuration from being diverted to being limited within the first 1000 \( \tau_A \) in the nonlinear computations. Over the first half of this period of time, the evolution of plasma current \( (I_p) \) and thermal energy is nearly identical in the axisymmetric and 3D computations. Figure 4 shows that discrepancies between the two computations develop over the second half of this period and increase thereafter, especially for the thermal energy. The asymmetric perturbations in the 3D computation achieve their maximum amplitude during the period \( 500 \tau_A \leq t \leq 1100 \tau_A \), and as evident from the multiple local maxima in the fluctuation spectra shown in Fig. 5, saturation is not a simple process. The distortion of the plasma cross-section during this early saturation phase, shown by plasma pressure in Fig. 6, indicates that the \( m = 3 \) perturbation dominates prior to \( t = 500 \tau_A \). However, the \( m = 2 \) perturbation dominates by \( t = 600 \tau_A \). Plots of the \( n = 1 \) component of pressure (not shown) support the finding that the dominant poloidal wavenumber

\[ \ldots \]
changes over time. The perturbations also alter the magnetic topology, producing magnetic islands near the edge of the distorting region of confinement and shrinking the volume of closed field lines. While the closed-flux region of the axisymmetric computation also decreases, the decrease in that case results from contact with the resistive wall, alone. This effect also occurs in the 3D computation, but chaotic scattering from the asymmetric perturbations is more significant, and anisotropic thermal conduction along the scattered field lines increases the rate of thermal energy loss. By \( t = 1000 \tau_A \), there are no closed flux surfaces remaining in the 3D computation. The rate of thermal energy loss is largest at this time, when the highest-temperature region loses closed-flux confinement.

![Figure 4](image_url)

Figure 4. Comparisons of a) plasma current and b) thermal energy evolution for the axisymmetric and 3D nonlinear computations.

![Figure 5](image_url)

Figure 5. Evolution of a) magnetic energy inside the resistive wall and b) kinetic energy, decomposed by toroidal Fourier wavenumber, from the 3D computation. Traces for all wavenumbers are plotted, except the large \( n = 0 \) component of magnetic energy; only those for \( n \leq 10 \) are labeled. For reference, the \( n = 0 \) magnetic field inside the resistive wall, excluding the vacuum toroidal field, starts with 2.9 normalized units of energy.
Figure 6. Color contours of plasma pressure overlaid with Poincaré surfaces of section from magnetic field-line tracing at a) $t = 473\tau_A$, b) $t = 595\tau_A$, and c) $t = 759\tau_A$. Each frame shows results from the 3D computation at toroidal angle $\phi = 0$.

As a description of VDE evolution in the absence of asymmetric instabilities, the axisymmetric computation provides information on the source of free energy for MHD activity in the 3D computation during its vertical transient. With the condition of $\tau_w << \tau_\eta$, contact with the wall scrapes-off the outer part of the equilibrium, while leaving the core relatively unchanged. Figure 7a shows the effect on the safety factor profile, and the most striking feature is the decrease in edge $q$-values over time. We surmise that the effect changes resonance conditions for different wavenumbers in the 3D computation. Moreover, the loss of the outer part of the $I(\psi)$ profile enhances $I'$, hence $\lambda$ in the edge. Figure 7b shows that the resulting edge-$\lambda$ is nearly as large as the central $\lambda$-value, considerably larger than the initial edge-$\lambda$. The narrowness of this edge-current layer implies free energy for current-gradient-driven MHD modes, as also noted in Ref. [21], hence the excitation of kink-type behavior, evidently followed by magnetic tearing, when asymmetries are allowed. Contact with the wall also increases the edge pressure gradient in the axisymmetric computation. For $q(a) > 1$, this edge pressure gradient would tend to drive ballooning, but no distinct high-$n$ activity appears in the 3D computation (Fig. 5). The loss of edge thermal confinement from the low-$n$ perturbations in the 3D computation precludes the development of a clear edge pressure pedestal, and ballooning does not arise.
Figure 7. Results from the nonlinear axisymmetric computation showing a) evolution of the safety factor profile and b) $\lambda = \mu_0 J_{\parallel}/B$ (color contours) and poloidal flux (lines) at $t = 1035 \tau_A$.

The range of $\psi$-values in a) is that of the closed flux in the initial state.

3.3 Current bump and distribution of current density

The separation of the two plasma current histories in Fig. 4a starts from the initial saturation of the asymmetric instabilities, which increases $I_p$ in the 3D case. Comparing Figs. 4 and 5a shows that the $I_p$ bump continues after $t = 1000 \tau_A$, which is when magnetic fluctuations and the rate of thermal energy loss are largest. This aspect is consistent with the discussion of the JET current spike in Fig. 29 of Ref. [25] and with the data for a major disruption in TFTR shown in Fig. 1 of Ref. [26]. Comparing the two frames in Fig. 8 shows that the parallel current density distribution, computed from the toroidally symmetric part of $B$, spreads while $I_p$ increases. The concentration of poloidal flux within the central-plasma region also decreases over this time, which can be observed from the decreased density of the equally spaced, poloidal-flux contours. The increase in $I_p$ simultaneous with the spreading of poloidal flux necessarily implies a reduction of inductance. This effect does not occur in the axisymmetric computation, where the distribution of poloidal flux within the plasma core remains relatively constant while the edge is removed by contact with the wall (Fig. 7b). Similar behavior has been noted in simulations of MHD activity excited by edge impurities for disruption mitigation [35].
Figure 8. Color contours of $\lambda = \mu_0 \langle J \cdot B \rangle / \langle B \rangle^2$ overlaid with contours of constant poloidal flux $\langle \psi \rangle$, where $\langle f \rangle$ indicates the toroidal average of $f$, at a) $t = 473 \tau_A$ and b) $t = 1373 \tau_A$ in the 3D computation. The uniform increments between contour levels in $\langle \psi \rangle$ are the same in Figs. 7b and 8.

Part of the change of poloidal flux can be attributed to resistive dissipation that is enhanced by the decreasing temperature during the TQ. However, a stronger effect results from the correlation of magnetic and flow-velocity fluctuations, the MHD dynamo effect $E_f \equiv -\langle \nabla \times \dot{B} \rangle$, where "fluctuation" refers to the toroidally asymmetric component of a field. Conceptually, this stems from mean-field theory of MHD for astrophysics [36] and was first appreciated for magnetic confinement in the context of magnetic relaxation in reversed-field pinches [37, 38]. It was later applied to analyses of driven spheromaks [39, 40], helicity injection in tokamaks [41] and spherical tokamaks [42-44], and the tokamak hybrid mode [45]. Averaging Faraday's law, $\partial \langle B \rangle / \partial t = -\nabla \times \langle E \rangle$, and considering low frequencies leads to the Poynting theorem for the symmetric field,

$$\frac{1}{2\mu_0} \frac{\partial}{\partial t} \langle B \rangle^2 + \frac{1}{\mu_0} \nabla \cdot \langle E \rangle \times \langle B \rangle = -\langle E \rangle \cdot \langle J \rangle . \quad (9)$$

The term $E_f \cdot \langle J \rangle$ is included in the right side of Eq. (9) and is part of a nonlinear magnetic energy transport process that acts through MHD fluctuations [41, 46]. Figure 9 shows this power density and the toroidal component of $E_f$ at $t = 759 \tau_A$, which is when the $I_p$ bump begins in the 3D computation. This power density is mostly positive near the core and negative in the edge, removing energy from $\langle B \rangle$ in the core and depositing it in the edge. This process spreads the current-density distribution. In addition, the fluctuation-induced loop voltage from $E_f \cdot \hat{\phi}$ redistributes poloidal flux; locations where $RE_{f \hat{\phi}}$ decreases in the flux-normal direction push
the poloidal flux distribution outward. We also note that the magnitude of the fluctuation-induced electric field shown in Fig. 9b is approximately 100 times that of $\eta \mathbf{J} \cdot \hat{\phi}$ on the magnetic axis at the start of the computation.

![Figure 9](image)

Figure 9. Color contours of a) the fluctuation induced power density $E_f \cdot \langle \mathbf{J} \rangle$ and b) $E_f \cdot \hat{\phi}$ at $t = 759 \tau_A$. Frame a) also has contour lines of $\langle \psi \rangle$ with the same spacing as in Figs. 7 and 8.

The consideration of correlated fluctuations helps describes the nonlinear effects of asymmetric instabilities on the toroidally averaged field, but it does not provide a full sense of their influence. Section 3.1 already noted that chaotic scattering extends over the entire plasma volume by the condition shown in Fig 8b. The local evaluation of $\lambda$ at this time, Fig. 10, identifies considerable spatial structure that mean-field relaxation analysis does not. The spatial structure includes current sheets and regions of anti-parallel current.
Figure 10. Contours of constant $\lambda = \mu_0 J_\parallel / B$, evaluated locally in the $\phi = 0$ plane at $t = 1373 \tau_A$ of the 3D computation.

Our computation of net forces on the resistive wall follows the analysis given by Pustovitov in Ref. [27]. Two essential properties used in the analysis are that 1) together, the plasma and resistive wall form an electrically isolated system and 2) plasma inertia is negligible for the timescale of the evolution. The net force on the wall can then be computed from a surface integral of magnetic stresses over the outside of the wall. The thin-shell approximation of our model actually necessitates the use of stresses, because current per unit cross-section area is infinitely large. Here, we check the consistency of our computed results with Pustovitov's prediction by starting from

$$F_j = \frac{1}{\mu_0} \hat{e}_j \cdot \left[ \int_{S_{in}} dS \left( BB - \frac{1}{2} B^2 \right) + \int_{S_{out}} dS \left( BB - \frac{1}{2} B^2 \right) \right],$$

i.e. $F_j = F_{in,j} + F_{out,j}$, where $\hat{e}_j$ is a Cartesian unit vector and the integral is broken into separate contributions over the inner and outer surfaces of the resistive wall. Noting that the net force acting on the plasma is $-\vec{F}_{in}$ and, therefore, the implication from property 2 of $\vec{F}_{in} \to 0$, we expect $F_j \to F_{out,j}$.

The net horizontal force, which only results with toroidal asymmetries, peaks at normalized amplitude of 0.08 at $t = 3000 \tau_A$ in the 3D computation (Fig. 11a). To provide a sense of scale, if this force was generated from $m = 1$, $n = 1$ tilting of the plasma current, Noll's relation [3], $F = \pi B_\phi I_p \delta z$, would imply that the toroidally asymmetric displacement would only be 0.011. Nonetheless, we can check whether Pustovitov's property 2 holds for this relatively weak force. We find that the largest instantaneous realization of $F_{in}/F_{out}$ is the value of $3 \times 10^{-2}$ that occurs briefly at $t = 600 \tau_A$. Using the maximum over time of each integral, we find
max\( (F_{in})/\max (F_{out}) = 5 \times 10^{-3} \). These ratios are larger than what is discussed in Ref. [27] for cases of large forces in JET, but even here they are consistent with the analysis.

Separating the horizontal components of force, Fig. 11b, shows that the orientation of the horizontal force vector changes over the simulated displacement event. This has been observed in previous MHD simulations of disruption [47], but effects outside the scope of MHD are likely needed to predict the rotation of forces.

Figure 11. Net forces acting on the resistive wall in the 3D computation. Frame a) shows the magnitude of net horizontal force from \( \bar{F}_{out} \) and \( \bar{F}_{in} \), and frame b) shows Cartesian components of \( \bar{F}_{out} \).

Comparing Figs. 5a and 11a shows that the largest horizontal force on the wall occurs much later than the peak of the asymmetric magnetic fluctuations. In fact, the force continues to increase while the energy in the \( n = 1 \) component of \( B \) decreases over \( 1000 \tau_A < t < 2200 \tau_A \). Figure 12 plots the \( \hat{y} \)-component of the force per unit area on the wall, which results from the jump in Maxwell stress for MHD,

\[
\frac{1}{\mu_0} \hat{n} \cdot \Delta \left( \frac{BB - I B^2}{2} \right) \hat{y},
\]

where \( \hat{n} \) is the unit outward normal along the wall. The maximum force density is larger at \( t = 3000 \tau_A \) than at \( t = 971 \tau_A \), despite the smaller internal fluctuation energy. During the intervening time, the discharge has moved somewhat closer to the lower divertor, and the magnetic perturbations have more time to penetrate the resistive wall. The distribution over the toroidal angle has also changed such that there is less cancellation when integrating over the inboard and outboard sides.
Figure 12. Force per unit area, $\hat{y}$-component shown, over the resistive wall at a) $t = 971\tau_A$ and b) $t = 3000\tau_A$. Horizontal axes covers the toroidal angle $\phi$, and vertical axes cover the geometric poloidal angle $\theta$ about $R = 1.575$, $Z = 0$ with $\theta = 0$ pointing radially outward. For reference, the angles $\theta = \pm 1$ rad point toward the outboard corners of the resistive wall, the angles $\theta = \pm 2.14$ rad point toward the inboard corners, and the lower divertor coil lies along $\theta = -1.79$ rad.

4. Discussion

Visco-resistive MHD modeling greatly simplifies the entirety of physics that influences disruptions in tokamak experiments. Approximations that directly influence the computational results presented above are the lack of radiation modeling, the Dirichlet boundary conditions on temperature, and the lack of runaway-electron (RE) effects. The modeled TQ resulting from local anisotropic thermal transport in the presence of changing magnetic topology is fast in our 3D computation, relative to the CQ. However, a realistic kinetic model for parallel heat transport, or even a larger value of $\chi_{||}$, would further separate the TQ and CQ timescales. Radiation from impurities is also expected to have an important role in the TQ, and as noted, it has not been modeled. The present computations rely on thermal conduction to the resistive wall, where the imposed Dirichlet condition maintains low temperature. Parallel conduction quickly cools open field-lines, leading to the thin halo region in our axisymmetric result. From Ref. [21] and our work in this area [31], we know that computed VDE evolution is sensitive to edge temperature, so more comprehensive modeling is needed. For example, boundary conditions for edge turbulence modeling have been developed using conditions at the magnetic pre-sheath entrance [48]. We are adapting this more realistic approach for our disruption computations [31], but because electrons are then largely insulated, it may also require a radiation model to expel electron heat that is conducted toward the surface. Finally, experimental CQs are often extended by the formation of RE beams [1, 25, 49]. We expect that practical computations in the near future will rely on reducing modeling of REs, such as the one developed in Ref. [50] and already applied in recent M3D simulations.

The possibility of kink-induced surface current, progressing in the direction opposite to that of the plasma current and traveling partly through conducting structures has been raised in Ref. [5]. While this wall-touching kink mode (WTKM) is predicted to be most problematic for the $m = 1$,
$n = 1$ mode, which arises with sufficiently small $q$-values, the physics of reversed surface currents also arises with other external modes [51, 52]. In Sect. 3.3, we note the existence of reversed parallel current density at the time of maximum-$I_p$, but the nonlinearly distorted current profile makes it difficult to relate the reversed parallel current density with any particular kink mode. However, at $t = 473\tau_A$ when the $m = 3$, $n = 1$ mode dominates, the external kink distortion is clearer. The $\lambda = -0.15$ isosurface in Fig. 13 is computed from the total field of the 3D computation at this time, and it lies along the outward bulge from the kink distortion. The existence of this helical current channel demonstrates that the physics of reversed surface currents is within the scope of resistive-MHD models using $T$-dependent resistivity to track an effective plasma surface.

Figure 13. Isosurfaces of $\lambda$ at values of $-0.15$ (violet) and $0.60$ (orange) from $t = 473\tau_A$ of the 3D computation.

5. Conclusions

This computational study of an asymmetric VDE in an idealized configuration considers a number of physical effects that are important for tokamak experiments. The 3D result reproduces distinct timescales for the TQ and CQ phases, despite the modest parallel heat conduction and absence of RE physics. The thermal energy collapse results from chaotic scattering of magnetic field lines associated with increasingly pervasive MHD modes and nonlinear coupling. In addition, the MHD activity generates nonlinear processes that redistribute the parallel current density. We argue that the effects on the symmetric components of $B$ and $J$ can be understood through analysis of the fluctuation-induced $E_f$ electric field, which is the MHD dynamo effect known from studies of current drive in RFPs, spheromaks, and other configurations. Because the fluctuations do not dissipate rapidly, the decrease in inductance and the resulting bump in plasma current that are associated with current-density spreading persist beyond the time when the rate of thermal energy loss is greatest. We also find that relative to the axisymmetric result, the spreading of current reduces the attractive force exerted by the current in the divertor coil. Thus, final termination of the plasma current through contact with the wall takes longer with the asymmetric distortions.
We have computed forces on the resistive wall by integrating the Maxwell stress over its surface. In the modeled case, the net horizontal force is not large relative to conditions of significant tilting of the toroidal current. Nonetheless, we find that the net force over the inner surface of the wall is much smaller than that over its outer surface. This is consistent with the analysis of Ref. [27], which argues that the plasma and wall must remain in approximate force balance on timescales of interest, so the net force can be accurately computed by integrating stress over the outer surface alone. Our force computation takes into account contributions from asymmetric conduction currents flowing from the plasma to the resistive wall, including any reversed currents of external-kink dynamics, demonstrated in Sect. 4, which underlie the WTKM theory [5]. While the computed scenario does not develop a large \( m = 1, n = 1 \) distortion, our results support the possibility of reproducing WTKM physics.

This work also supports the prospect of using Eulerian-frame computation for simulating disruptions. We expect Lagrangian and other moving-frame approaches to be more efficient in cases where the plasma torus remains intact. However, instances of significant distortion and magnetic topology change, such as what our model case produces, would tangle Lagrangian meshes or at least remove many of the computational benefits of moving meshes. Nonetheless, 3D computations, such as the one presented here, are computationally intensive, and improved solver efficiencies and better use of recent computer hardware developments are needed to make these computations more practical. Predictive simulations of asymmetric VDEs are also expected to require detailed representations of external conductors, such as what has been developed for resistive-wall mode studies [53], and the physical model developments discussed in Section 4.

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